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Perturbative QCD Analysis of B Meson Decays

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Abstract

Resummation of large QCD radiative corrections, including leading and next-to-leading logarithms, in pion electromagnetic form factor is reviewed. Similar formalism is applied to exclusive processes involving heavy mesons, and leads to Sudakov suppression for the semi-leptonic decay $B \rightarrow \pi l \nu$. It is found that, with the inclusion of Sudakov effects, perturbative QCD analysis of this decay is possible for the energy fraction of the pion above 0.3. By combining predictions from the soft pion theorems, we estimate that the upper limit of the KM matrix element $|V_{ub}|$ is roughly 0.003.

1. Introduction

It has been shown that perturbative QCD (PQCD) is applicable to exclusive processes such as elastic hadron form factors for energy scale higher than few GeV [1]. The enlargement of the range of the applicability from much higher energies [2] down to this low scale is due to the inclusion of transverse momentum dependence into factorization theorems. This dependence appears in an exponential factor, which arises from the resummation of large radiative corrections, and leads to Sudakov suppression for elastic scattering of isolated colored quarks. Detailed derivation of Sudakov factors for hadron-hadron Landshoff scattering refers to [3]. Similar expressions have been obtained and employed in the PQCD analysis of pion and proton form factors [1, 4, 5, 6], pion Compton scattering [7] and other exclusive processes [8]. Resulting predictions from this modified version of factorization theorems have been examined and found to be dominated by perturbative contributions.

All the above analyses of large corrections involves only light hadrons. In this paper we shall extend the formalism to exclusive processes containing both light and heavy mesons, such as B meson decays, and organize their Sudakov corrections to all orders. An important work in the study of the standard model is to determine the mixing angles in the Cabibbo-Kobayashi-Maskawa (KM) matrix. The decay $K \rightarrow \pi l \nu$ contains the information of the matrix element $|V_{us}|$, and chiral symmetry provides a precise method to study this process [9]. Similarly, $|V_{cb}|$ is determined by exploring the $B \rightarrow D l \nu$ decay, for which heavy quark symmetry is an appropriate tool [10]. As to $|V_{ub}|$, which can be measured reliably from the $B \rightarrow \pi l \nu$ decays [11], neither of the above theories is proper.

Recently, an analysis of the semi-leptonic decay $B \rightarrow \pi l \nu$ based on the heavy quark effective theory (HQET) has been performed by Burdman *et al.* [12]. They have determined the normalization of form factors involved in the

process in terms of the soft pion relations, and given the ratio of these form factors to the corresponding ones in the $D \rightarrow \pi l \nu$ decay. However, explicit evaluation of this process is not yet successful. In this paper we shall show that, by incorporating Sudakov effects, PQCD is indeed applicable to the semi-leptonic B meson decays as the pion carries away more than a quarter of its maximum energy. The differential decay rate is then obtained. By combining our predictions with those from the soft pion theorems, which have been derived in the framework of HQET [12], the total decay rate and the branching ratio of this process are estimated. Comparing this estimation with experimental data, we find that the upper limit of the KM matrix element $|V_{ub}|$ is roughly 0.003. PQCD is then able to complement HQET in the analysis of heavy meson decays. Our results can be easily generalized to perturbative analysis of other heavy-to-light transitions.

We find that factorization theorems are successful for this semi-leptonic decay especially when the pion is energetic. In this case non-perturbative approaches such as the soft pion theorems and QCD sum rules are not applicable. The first attempt to apply the modified perturbative formalism to the analysis of the $B \rightarrow \pi l \nu$ decay has been made by Akhoury *et al.* [13]. However, they did not consider the transverse momentum dependence, and have obtained predictions which are drastically different from ours. As claimed in [13], the hard gluon involved in the decay process, with Sudakov effects taken into account, is off-shell at most by an amount of $1.4\Lambda_{\text{QCD}}m_b$, m_b being the b quark mass. In our approach the hard gluon is off-shell roughly by $8\Lambda_{\text{QCD}}m_b$, and therefore, the perturbative analysis is more reliable.

The resummation of Sudakov logarithms for the pion form factor is reviewed in section 2. The same formalism is then applied to semi-leptonic B meson decays in section 3, where the full expression for the Sudakov factor including leading and next-to-leading logarithms is given. Section 4 contains numerical analysis of the modified factorization formulas for the relevant form factors and the differential decay rate of $B \rightarrow \pi l \nu$. Section 5 is our

conclusion.

2. The Pion Form Factor

First, we review the modified factorization formula for a simple light-to-light process, the pion electromagnetic form factor, to lowest order of coupling constant α_s , which is expressed as the convolution of wave functions and a hard scattering amplitude. We then investigate radiative corrections to this formula, and explain how they are absorbed into the above convolution factors. The first step is to find out the leading momentum regions of radiative corrections, from which important contributions to loop integrals arise. There are two types of important contributions: collinear, when the loop momentum is parallel to the incoming or outgoing pion momentum, and soft, when the loop momentum is much smaller than the momentum transfer Q^2 of the process. Here, Q^2 is assumed to be large and serves as an ultraviolet cutoff of loop integrals. We associate small amount of transverse momenta \mathbf{k}_T with the partons in the above factorization picture, which is taken as an infrared cutoff.

Each type of these important contributions gives a large single logarithm. They may overlap to give a double (leading) logarithm in some cases. These large logarithms, appearing in a product with α_s , must be organized in order not to spoil the perturbative expansion. It is known that single logarithms can be summed to all orders using renormalization group methods, and double logarithms must be organized by the technique developed in [14]. We shall work in axial gauge $n \cdot A = 0$, in which the resummation technique is developed easily, n being the gauge vector and A the gauge field.

The diagrams shown in fig. 1 represent the $O(\alpha_s)$ radiative corrections to the basic factorization of the pion form factor, which contain large logarithms mentioned above. In axial gauge the two-particle reducible diagrams, like

figs. 1a and 1b, have double logarithms from the overlap of collinear and soft enhancements, while the two-particle irreducible corrections, like figs. 1c and 1d, contain only single soft logarithms. This distinction is consistent with the physical picture: two partons moving in the same direction can interact with each other through collinear or soft gluons, while those moving apart from each other can interact only through soft gluons. Below we shall concentrate on reducible corrections, and demonstrate how they are summed into the Sudakov factor.

A careful analysis shows that soft enhancements cancel between figs. 1a and 1b, as well as between 1c and 1d, in the asymptotic region with $b \rightarrow 0$, b being the conjugate variable to \mathbf{k}_T . Therefore, reducible corrections are dominated by collinear enhancements, and can be absorbed into the pion wave function, which involves similar dynamics. Irreducible corrections, due to the cancellation of their soft divergences, are then absorbed into the hard scattering amplitude. Hence, the factorization picture still holds at least asymptotically after radiative corrections are included. The above cancellation of soft divergences is closely related to the universality of wave functions. For large b , double logarithms are present and the resummation technique must be implemented.

Based on the above reasoning, the factorization formula for the pion form factor in the b space is written as [1],

$$F_\pi(Q^2) = \int_0^1 dx_1 dx_2 \int \frac{d^2\mathbf{b}}{(2\pi)^2} \mathcal{P}(x_2, b, P_2, \mu) \times H(x_1, x_2, b, Q, \mu) \mathcal{P}(x_1, b, P_1, \mu), \quad (1)$$

where μ is the factorization and renormalization scale, and b is the separation between the two valence quarks. P_1 and P_2 are momenta of the incoming and outgoing pions, respectively. Here we choose the Breit frame, in which $P_1^+ = P_2^- = Q/\sqrt{2}$ and all other components of P 's vanish, Q^2 being the momentum transfer, $Q^2 = -(P_1 - P_2)^2$. Note that eq. (1) depends only on a

single b , because the virtual quark line involved in the hard scattering H is thought of as far from mass shell, and is shrunk to a point [1]. The pion wave function \mathcal{P} includes all leading logarithmic enhancements at large b . The basic idea of the resummation technique is as follows. If the double logarithms appear in an exponential form $\mathcal{P} \sim \exp[-\text{const.} \times \ln Q \ln(\ln Q / \ln b)]$, the task will be simplified by studying the derivative of \mathcal{P} , $d\mathcal{P}/d\ln Q = C\mathcal{P}$. It is obvious that the coefficient C contains only large single logarithms, and can be treated by renormalization group methods. Therefore, working with C one reduces the double-logarithm problem to a single-logarithm problem.

The two invariants appearing in \mathcal{P} are $P \cdot n$ and n^2 , and by the scale invariance of n in the gluon propagator,

$$N^{\mu\nu}(q) = \frac{-i}{q^2} \left(g^{\mu\nu} - \frac{n^\mu q^\nu + q^\mu n^\nu}{n \cdot q} + n^2 \frac{q^\mu q^\nu}{(n \cdot q)^2} \right), \quad (2)$$

\mathcal{P} can only depend on a single large scale $\nu^2 = (P \cdot n)^2/n^2$. It is then easy to show that the differential operator $d/d\ln Q$ can be replaced by d/dn :

$$\frac{d}{d\ln Q} \mathcal{P} = -\frac{n^2}{P \cdot n} P^\alpha \frac{d}{dn^\alpha} \mathcal{P}. \quad (3)$$

The motivation for this replacement is that the momentum P flows through both quark and gluon lines, but n appears only on gluon lines. The analysis then becomes easier by studying the n , instead of P , dependence.

Applying d/dn_α to the gluon propagator, we get

$$\frac{d}{dn_\alpha} N^{\mu\nu} = -\frac{1}{q \cdot n} (N^{\mu\alpha} q^\nu + N^{\nu\alpha} q^\mu). \quad (4)$$

The momentum q that appears at both ends of a gluon line is contracted with the vertex, where the gluon attaches. After adding together all diagrams with different differentiated gluon lines and using the Ward identity, we arrive at the differential equation of \mathcal{P} as shown in fig. 2a, in which the square vertex represents

$$gT^a \frac{n^2}{P \cdot n q \cdot n} P_\alpha,$$

T^a being the color matrix. An important feature of the square vertex is that the gluon momentum q does not lead to collinear enhancements because of the nonvanishing n^2 . The leading regions of q are then soft and ultraviolet, in which fig. 2a can be factorized according to fig. 2b at lowest order. The part on the left-hand side of the dashed line is exactly \mathcal{P} , and that on the right-hand side is assigned to the coefficient C introduced before.

Therefore, we need a function \mathcal{K} to organize the soft enhancements from the first two diagrams in fig. 2b, and \mathcal{G} for the ultraviolet divergences from the other two diagrams. The soft subtraction employed in \mathcal{G} is to avoid double counting. Generalizing the above two functions to all orders, we derive the differential equation of \mathcal{P} ,

$$\frac{d}{d \ln Q} \mathcal{P}(x, b, P, \mu) = \left[2\mathcal{K}(b\mu) + \frac{1}{2} \mathcal{G}(x\nu/\mu) + \frac{1}{2} \mathcal{G}((1-x)\nu/\mu) \right] \times \mathcal{P}(x, b, P, \mu) . \quad (5)$$

The functions \mathcal{K} and \mathcal{G} have been calculated to one loop, and the single logarithms have been organized to give their evolutions in b and Q , respectively [3]. They possess individual ultraviolet poles, but their sum $\mathcal{K} + \mathcal{G}/2$ is finite such that Sudakov logarithms are renormalization-group invariant.

Substituting the expressions for \mathcal{K} and \mathcal{G} into eq. (5), we obtain the solution

$$\mathcal{P}(x, b, P, \mu) = \exp \left[- \sum_{\xi=x, 1-x} s(\xi, b, Q) \right] \mathcal{P}(x, b, \mu) . \quad (6)$$

The exponent s , grouping the double logarithms in \mathcal{P} , is expressed in terms of the variables

$$\begin{aligned} \hat{q} &\equiv \ln [\xi Q / (\sqrt{2}\Lambda)] \\ \hat{b} &\equiv \ln(1/b\Lambda) \end{aligned} \quad (7)$$

as [1]

$$s(\xi, b, Q) = \frac{A^{(1)}}{2\beta_1} \hat{q} \ln \left(\frac{\hat{q}}{\hat{b}} \right) + \frac{A^{(2)}}{4\beta_1^2} \left(\frac{\hat{q}}{\hat{b}} - 1 \right) - \frac{A^{(1)}}{2\beta_1} (\hat{q} - \hat{b})$$

$$\begin{aligned}
& - \frac{A^{(1)}\beta_2}{16\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{b})+1}{\hat{b}} - \frac{\ln(2\hat{q})+1}{\hat{q}}\right] \\
& - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right) \\
& - \frac{A^{(1)}\beta_2}{32\beta_1^3}\left[\ln^2(2\hat{q}) - \ln^2(2\hat{b})\right]
\end{aligned} \tag{8}$$

with $\Lambda \equiv \Lambda_{QCD}$. The coefficients $A^{(i)}$ and β_i are

$$\begin{aligned}
\beta_1 &= \frac{33-2n_f}{12}, \quad \beta_2 = \frac{153-19n_f}{24}, \\
A^{(1)} &= \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1\ln\left(\frac{e^\gamma}{2}\right),
\end{aligned} \tag{9}$$

where $n_f = 3$ in this case is the number of quark flavors and γ is the Euler constant. To derive eq. (8), the choice of the gauge vector $n^\mu \propto (P_1 + P_2)^\mu$ has been made.

The function $\mathcal{P}(x, b, \mu)$ and H still contain single logarithms from ultraviolet divergences, which need to be summed using their renormalization group equations [3]:

$$\begin{aligned}
\mathcal{D}\mathcal{P}(x, b, \mu) &= -2\gamma_q\mathcal{P}(x, b, \mu) \\
\mathcal{D}H(x_i, b, Q, \mu) &= 4\gamma_q H(x_i, b, Q, \mu),
\end{aligned} \tag{10}$$

with

$$\mathcal{D} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}. \tag{11}$$

$\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension in axial gauge. Solving eq. (10), the large- b asymptotic behavior of \mathcal{P} can be summarized as

$$\begin{aligned}
\mathcal{P}(x, b, P, \mu) &= \exp \left[- \sum_{\xi=x, 1-x} s(\xi, b, Q) - 2 \int_{1/b}^\mu \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(g(\bar{\mu})) \right] \\
&\times \mathcal{P}(x, b, 1/b),
\end{aligned} \tag{12}$$

where

$$\mathcal{P}(x, b, 1/b) = \phi(x, b) + O(\alpha_s(1/b)) . \quad (13)$$

The evolution of \mathcal{P} in b , denoted by $O(\alpha_s)$, has been neglected. The b dependence in ϕ , corresponding to the intrinsic transverse momentum dependence of the pion wave function [6, 15], will not be considered here.

Similarly, the renormalization group analysis applied to H gives

$$\begin{aligned} H(x_i, b, Q, \mu) &= \exp \left[-4 \int_{\mu}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(g(\bar{\mu})) \right] \\ &\times H(x_i, b, Q, t) , \end{aligned} \quad (14)$$

where t is the largest mass scale involved in the hard scattering,

$$t = \max(\sqrt{x_1 x_2} Q, 1/b) . \quad (15)$$

The scale $\sqrt{x_1 x_2} Q$ is associated with the longitudinal momentum of the hard gluon and $1/b$ with its transverse momentum. Combining all the exponents derived above, we obtain the lowest-order expression for the pion form factor,

$$\begin{aligned} F_{\pi}(Q^2) &= 16\pi\mathcal{C}_F \int_0^1 dx_1 dx_2 \phi(x_1) \phi(x_2) \int_0^{\infty} b db \alpha_s(t) K_0(\sqrt{x_1 x_2} Q b) \\ &\times \exp[-S(x_1, x_2, b, Q)] , \end{aligned} \quad (16)$$

where the complete Sudakov logarithms are given by

$$S(x_1, x_2, b, Q) = \sum_{i=1}^2 [s(x_i, b, Q) + s(1 - x_i, b, Q)] - \frac{2}{\beta_1} \ln \frac{\hat{t}}{b} , \quad (17)$$

with $\hat{t} = \ln(t/\Lambda)$. \mathcal{C}_F is the color factor defined by $\text{tr}(T^a T^a) = N_c \mathcal{C}_F$, N_c being the number of colors. K_0 is the modified Bessel function of order zero, which is the Fourier transform of the gluon propagator to the b space.

Variation of S with b has been displayed in [1], which shows a strong falloff in the large b region, and vanishes for $b > 1/\Lambda$. Hence, Sudakov

suppression selects components of the pion wave functions with small spatial extent b , and makes the hard scattering more perturbative. If b is small, α_s with its argument set to t as in eq. (15) will be small, regardless of the values of the x 's. When b is large and $x_1 x_2 Q^2$ is small, α_s is still large. However, the Sudakov factor in eq. (16) strongly suppresses this region. Since the main contributions to the factorization formula come from the small b , or short-distance, region, the perturbation theory becomes relatively self-consistent. It is also easy to show that the modified factorization formula including Sudakov corrections reduces to the standard one as $b \rightarrow 0$, where the important logarithms diminish.

3. The Decay $B \rightarrow \pi l \nu$

We now generalize the above analysis of the pion form factor to the exclusive processes involving both light and heavy mesons. In particular, we work with the semi-leptonic decay $B \rightarrow \pi l \nu$. PQCD is appropriate to this process when the pion is energetic, because the b quark mass provides a large scale. We shall analyze the leading regions of radiative corrections to such a process, and derive the Sudakov factor including both leading and next-to-leading logarithms. The amplitude of this decay is written as

$$A(P_1, P_2) = \frac{G_F}{\sqrt{2}} V_{ub} \bar{\nu} \gamma_\mu (1 - \gamma_5) l \langle \pi(P_2) | \bar{b} \gamma^\mu u | B(P_1) \rangle, \quad (18)$$

where the four-fermion interaction with the Fermi coupling constant G_F has been inserted. P_1 and P_2 are momenta of the B meson and the pion, respectively. We start with the lowest-order factorization for the matrix element $M^\mu = \langle \pi(P_2) | \bar{b} \gamma^\mu u | B(P_1) \rangle$ with an exchanged hard gluon as shown in fig. 3, the left-hand side being the B meson at rest and the right-hand side a fast-recoiling pion. The heavy b quark is represented by a bold line. The symbol

\times denotes the electroweak vertex with the KM matrix element V_{ub} , from which a lepton pair emerges.

Parton momenta are assigned as in fig. 3. The b quark carries $P_1 - k_1$, and the accompanying light quark carries k_1 . They satisfy the on-shell conditions $(P_1 - k_1)^2 \approx m_b^2$, $P_1^2 = m_B^2$ and $k_1^2 \approx 0$, m_B being the B meson mass. In the Breit frame P_1 has the components $P_1^+ = P_1^- = m_B/\sqrt{2}$ and vanishing transverse components. k_1 may have a large minus component with small amount of transverse components \mathbf{k}_{1T} , which will serve as the infrared cutoff of Sudakov corrections below. The assignment of parton momenta on the pion side is similar to that in the case of the pion form factor as shown in fig. 3. The large component of P_2 is $P_2^+ = \eta m_B/\sqrt{2}$, η being related to the energy fraction of the pion by $P_2^0 = \eta m/2$. The physical range of η is $0 \leq \eta \leq 1$, since the pion carries away at most half of the rest energy of the B meson. The transverse momentum associated with the valence quarks of the pion is denoted by \mathbf{k}_{2T} . The invariant mass of the lepton-neutrino pair produced in this decay is given by $m_l^2 = (P_1 - P_2)^2 = (1 - \eta)m_B^2$.

We then consider radiative corrections to the basic factorization for the heavy-to-light transition. The essential step is again to find out the leading regions of radiative corrections in axial gauge. For reducible corrections on the pion side, the conclusion is the same as before: they produce double logarithms with soft ones cancelled in the asymptotic region, and can be absorbed into the pion wave function, which give rise to the evolution of the wave function. Irreducible corrections, with an extra gluon connecting a quark in the pion and a quark in the B meson, give only soft divergences, which also cancel asymptotically. Hence, they are absorbed into the hard scattering amplitude.

On the left-hand side, three diagrams showing the $O(\alpha_s)$ reducible radiative corrections, are exhibited in fig. 4. Fig. 4a, giving the self-energy correction to the massive b quark, produces only soft enhancements, and is thus not leading. If k_1^- is small, collinear divergences in figs. 4b and 4c, which

arise from the loop momentum with a large component parallel to k_1 , will not be pinched, and they also give only soft enhancements. This is consistent with the physical picture that the soft light quark can not interact with the heavy quark through a fast moving gluon. However, from the B meson wave functions given in section 4, it is observed that there is substantial probability of finding the light quark with k_1^- of order m_b , even though the wave functions peak at small k_1^- . Therefore, figs. 4b and 4c contribute collinear enhancements. Note that fig. 4b gives soft divergences which are not completely cancelled by those from figs. 4a and 4c even in the asymptotic region. In conclusion, figs. 4b and 4c indeed contain double logarithms, which must be organized by the resummation technique.

Since the collinear enhancements on the B meson side are less important due to the suppression of the wave functions, reducible corrections are basically dominated by soft enhancements, and can be absorbed into the B meson wave function, which is also dominated by soft dynamics. This absorption of reducible corrections is similar to that on the pion side, where reducible corrections are dominated by collinear enhancements. Therefore, the factorization picture in fig. 3 still holds after radiative corrections are included.

With the above reasoning, we can write down the factorization formula for the decay $B \rightarrow \pi l \nu$ in the transverse configuration space,

$$\begin{aligned}
M^\mu &= \int_0^1 dx_1 dx_2 \int \frac{d^2 \mathbf{b}_1}{(2\pi)^2} \frac{d^2 \mathbf{b}_2}{(2\pi)^2} \mathcal{P}_\pi(x_2, \mathbf{b}_2, P_2, \mu) \\
&\quad \times \tilde{H}^\mu(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, m, \mu) \mathcal{P}_B(x_1, \mathbf{b}_1, P_1, \mu) , \quad (19)
\end{aligned}$$

where both the pion and B meson wave functions, \mathcal{P}_π and \mathcal{P}_B , contain leading double logarithms. Here we have to introduce two b 's, because the virtual quark line in the hard scattering may not be far from mass shell, and can not be shrunk to a point. This is contrary to the case of the pion form factor, and detailed explanation will be given later. Therefore, we need b_1 to denote

the separation between the two valence quarks of the B meson, and b_2 for the pion. The approximation $m_b \approx m_B = m$ has been made. The momentum fraction x_1 is defined as k_1^-/P_1^- . \tilde{H}^μ is the Fourier transform of the hard scattering amplitude derived from fig. 3, whose explicit expression will be given in section 4. The resummation of the double logarithms in \mathcal{P}_π has been performed in the previous section. Here we quote the results directly with Q set to ηm and n_f set to 4 [13]. In the below we shall concentrate on \mathcal{P}_B .

There are two major difficulties in summing up the double logarithms in figs. 4b and 4c. Firstly, fig. 4a, which gives only single logarithms, must be excluded. Secondly, there are many invariants that can be constructed from P_1 , k_1 and n such as P_1^2 , $P_1 \cdot k_1$, $P_1 \cdot n$, $k_1 \cdot n$ and n^2 , which are involved in \mathcal{P}_B . In the pion case the number of invariants is much less. There are only $P \cdot n$ and n^2 , and the resummation is thus simpler. The fact that \mathcal{P}_B contains many invariants fails the technique of replacing d/dm by d/dn as introduced in section 3, because in this case some large scales like P_1^2 can not be related to n .

However, the above difficulties can be overcome by applying the eikonal approximation to the heavy quark line as shown in fig. 5. In the collinear region with the loop momentum parallel to k_1 and in the soft region, the b quark line can be replaced by an eikonal line:

$$\frac{(\not{P}_1 - \not{k}_1 + \not{q} + m)\gamma^\alpha}{(P_1 - k_1 + q)^2 - m^2} \approx \frac{(P_1 - k_1)^\alpha}{(P_1 - k_1) \cdot q} + R, \quad (20)$$

where the remaining part R either vanishes as contracted with the matrix structure of the B meson wave function, or is less leading. The factor $1/[(P_1 - k_1) \cdot q]$ is associated with the eikonal propagator, and the numerator $(P_1 - k_1)^\alpha$ is absorbed into the vertex, where a gluon attaches to the eikonal line. The physics involved in this approximation is that a soft gluon or a gluon moving parallel to k_1 can not explore the details of the b quark, and its dynamics can be factorized. The idea is similar to that employed in HQET.

An explicit evaluation of radiative corrections confirms this approximation. By this means, the first difficulty is resolved, because self-energy diagrams of an eikonal line are excluded by definition [16].

The eikonal approximation also reduces the number of large invariants involved in \mathcal{P}_B . We have the scale invariance in $P_1 - k_1 \approx P_1$ as shown by Feynman rules for an eikonal line in eq. (20), which corresponds to the flavor symmetry in HQET, in addition to the scale invariance in n . Hence, P_1 does not lead to a large scale, and the only remaining large scale is k_1^- , which must appear in the ratios $(k_1 \cdot n)^2/n^2$ and $(k_1 \cdot P_1)^2/P_1^2$. An explicit lowest-order investigation shows that the second scale $(k_1 \cdot P_1)^2/P_1^2$ in fact does not exist. Therefore, with the eikonal approximation the problem is simplified to the one in analogy with the light-meson case. Now \mathcal{P}_B depends only on the single large scale $\nu'^2 = (k_1 \cdot n)^2/n^2$, and d/dk_1^- can be replaced by d/dn .

Following the same procedures as in section 2, the differential equation of \mathcal{P}_B is derived as

$$\frac{d}{d \ln m} \mathcal{P}_B = \frac{d}{d \ln k_1^-} \mathcal{P}_B = \left[\mathcal{K}(b\mu) + \frac{1}{2} \mathcal{G}(\nu'/\mu) \right] \mathcal{P}_B, \quad (21)$$

where the lowest-order \mathcal{K} can be obtained from fig. 6a, and \mathcal{G} from fig. 6b, with the square vertex representing

$$gT^a \frac{n^2}{k_1 \cdot nq \cdot n} k_1^\alpha.$$

Note the absence of the diagrams corresponding to self-energy corrections to the eikonal line.

Comparing fig. 6a with 2a, we find that the evaluation of \mathcal{K} for the B meson is similar to that for the pion except the third diagram. This extra diagram is finite without ultraviolet and infrared divergences, as justified by its integral

$$g^2 \mathcal{C}_F \int \frac{d^4 q}{(4\pi)^4} \frac{n^2 k_1^\alpha P_1^\beta}{k_1 \cdot nq \cdot n P_1 \cdot q} \frac{i N_{\alpha\beta}}{q^2} e^{i q_T \cdot \mathbf{b}}. \quad (22)$$

Hence, it does not spoil the renormalization-group invariance of the Sudakov logarithms. For the specific choice $n \propto P_1$ as in the case of the pion form factor, it is easy to show that (22) vanishes. Therefore, the functions \mathcal{K} and \mathcal{G} for the B meson are in fact the same as those for the pion.

Substituting the expressions of \mathcal{K} and \mathcal{G} into eq. (21), we obtain the solution

$$\mathcal{P}_B(x_1, b_1, P_1, \mu) = \exp[-s(x_1, b_1, m)] \mathcal{P}_B(x_1, b_1, \mu) , \quad (23)$$

where the exponent s is given by eq. (8) but with $n_f = 4$ [13]. Summing up single logarithms in $\mathcal{P}_B(x_1, b_1, \mu)$, eq. (23) becomes

$$\begin{aligned} \mathcal{P}_B(x_1, b_1, P_1, \mu) = & \exp \left[-s(x_1, b_1, m) - 2 \int_{1/b_1}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(g(\bar{\mu})) \right] \\ & \times \phi_B(x_1, b_1) + O(\alpha_s(1/b_1)) , \end{aligned} \quad (24)$$

where the anomalous dimension γ_q is the same as before. Again, the evolution of \mathcal{P}_B in b_1 and the intrinsic b_1 dependence of the wave function ϕ_B will be neglected below. Including the summation of single logarithms in \tilde{H}^μ and the results from \mathcal{P}_π , we obtain the complete Sudakov exponent

$$\begin{aligned} S(x_i, b_i, \eta, m) = & s(x_1, b_1, m) + s(x_2, b_2, \eta m) + s(1 - x_2, b_2, \eta m) \\ & - \frac{1}{\beta_1} \left(\ln \frac{\hat{t}}{\hat{b}_1} + \ln \frac{\hat{t}}{\hat{b}_2} \right) . \end{aligned} \quad (25)$$

The variable \hat{b}_i is defined as in eq. (7), and t is the largest scale involved in the hard scattering, which will be discussed in section 4. Similar expression to eq. (25) for the pion form factor, which also involves two b 's, has been obtained in [4].

4. Numerical Results

Having derived the Sudakov factor for the process $B \rightarrow \pi l \nu$, we now evaluate its decay rate, and examine how much contribution comes from the perturbative region with small b_i . The explicit formula for lowest-order $H^{\mu(a)}$ from fig. 3a is written as

$$\begin{aligned}
H^{\mu(a)} &= \text{tr} \left[\gamma_\alpha \frac{\gamma_5 \not{P}_2}{\sqrt{2N_c}} \gamma^\mu \frac{\not{P}_1 - x_2 \not{P}_2 + \not{k}_{2T} + m}{(P_1 - x_2 P_2 + \mathbf{k}_{2T})^2 - m^2} \gamma^\alpha \frac{(\not{P}_1 + m)\gamma_5}{\sqrt{2N_c}} \right] \\
&\quad \times \frac{-g^2 N_c \mathcal{C}_F}{(x_2 P_2 - k_1 + \mathbf{k}_{2T})^2} \\
&= \frac{4(1 + x_2 \eta) g^2 \mathcal{C}_F m^2}{[x_2 \eta m^2 + \mathbf{k}_{2T}^2][x_1 x_2 \eta m^2 + (\mathbf{k}_{1T} - \mathbf{k}_{2T})^2]} P_2^\mu, \tag{26}
\end{aligned}$$

where the factors $\gamma_5 \not{P}_2/\sqrt{2N_c}$ and $(\not{P}_1 + m)\gamma_5/\sqrt{2N_c}$ come from the matrix structures of the pion and B meson wave functions, respectively [13]. The relation $k_1^- = x_1 m/\sqrt{2}$ has been inserted. In the second expression the \mathbf{k}_{2T} dependence in the fermion propagator is not neglected. For the pion form factor the corresponding transverse momentum dependence is negligible, because there is a factor x_2 in the numerator, which cancels the singularity from $x_2 \rightarrow 0$ in the denominator. However, for the B meson decays, due to the massiveness of the b quark, such a cancellation does not appear as shown in eq. (26). To ensure that the virtual quark be part of the hard scattering, \mathbf{k}_{2T} must be retained. Therefore, with the inclusion of transverse momenta, we need not to perform the subtraction of an on-shell fermion propagator from the hard scattering as in [13].

Similarly, the expression for lowest-order $H^{\mu(b)}$ is given by

$$\begin{aligned}
H^{\mu(b)} &= \text{tr} \left[\gamma_\alpha \frac{\gamma_5 \not{P}_2}{\sqrt{2N_c}} \gamma^\alpha \frac{\not{P}_2 + \not{k}_1}{(P_2 + k_1)^2} \gamma^\mu \frac{(\not{P}_1 + m)\gamma_5}{\sqrt{2N_c}} \right] \\
&\quad \times \frac{-g^2 N_c \mathcal{C}_F}{(x_2 P_2 - k_1 + \mathbf{k}_{2T})^2} \\
&= \frac{4g^2 \mathcal{C}_F x_1 \eta m^2}{[x_1 \eta m^2 + \mathbf{k}_{1T}^2][x_1 x_2 \eta m^2 + (\mathbf{k}_{1T} - \mathbf{k}_{2T})^2]} P_1^\mu \\
&\quad - \frac{4g^2 \mathcal{C}_F x_1 m^2}{[x_1 \eta m^2 + \mathbf{k}_{1T}^2][x_1 x_2 \eta m^2 + (\mathbf{k}_{1T} - \mathbf{k}_{2T})^2]} P_2^\mu. \tag{27}
\end{aligned}$$

To derive the second expression, we have replaced k_1^μ by

$$\frac{P_2 \cdot k_1}{P_1 \cdot P_2} P_1^\mu + \left(\frac{P_1 \cdot k_1}{P_1 \cdot P_2} - \frac{2}{\eta} \frac{P_2 \cdot k_1}{P_1 \cdot P_2} \right) P_2^\mu . \quad (28)$$

Here the transverse momentum \mathbf{k}_{1T} in the fermion propagator is negligible, because the singularity from $x_1 \rightarrow 0$ is removed by the numerator. However, we will keep it for consistency.

Performing the Fourier transform of eqs. (26) and (27), the matrix element M^μ can be written as

$$M^\mu = f_1 P_1^\mu + f_2 P_2^\mu , \quad (29)$$

in which the factorization formulas for the form factors f_1 and f_2 are given by

$$\begin{aligned} f_1 = & 16\pi\mathcal{C}_F m^2 \int_0^1 dx_1 dx_2 \int_0^\infty db_1 db_2 db_3 \phi_B(x_1) \phi_\pi(x_2) \\ & \times x_1 \eta h(x_1, x_2, b_1, b_2, \eta, m) \exp[-S(x_i, b_i, \eta, m)] , \end{aligned} \quad (30)$$

and

$$\begin{aligned} f_2 = & 16\pi\mathcal{C}_F m^2 \int_0^1 dx_1 dx_2 \int_0^\infty db_1 db_2 db_3 \phi_B(x_1) \phi_\pi(x_2) \\ & \times [-x_1 h(x_1, x_2, b_1, b_2, \eta, m) + (1 + x_2 \eta) h(x_2, x_1, b_2, b_1, \eta, m)] \\ & \times \exp[-S(x_i, b_i, \eta, m)] , \end{aligned} \quad (31)$$

respectively, with

$$\begin{aligned} h(x_1, x_2, b_1, b_2, \eta, m) = & \alpha_s(t) K_0(\sqrt{x_1 x_2 \eta} m b_2) \\ & \times [\theta(b_1 - b_2) K_0(\sqrt{x_1 \eta} m b_1) I_0(\sqrt{x_1 \eta} m b_2) \\ & + \theta(b_2 - b_1) K_0(\sqrt{x_1 \eta} m b_2) I_0(\sqrt{x_1 \eta} m b_1)] . \end{aligned} \quad (32)$$

I_0 is the modified Bessel function of order zero. From eqs. (26) and (27), we choose the largest scale t associated with the hard gluon as

$$t = \max(\sqrt{x_1 x_2 \eta} m, 1/b_1, 1/b_2) . \quad (33)$$

Basically, the above expressions for f_i are similar to that for the pion form factor but with different Sudakov logarithms.

To evaluate f_i , we consider the following two models of $\phi_B(x)$, which have been adopted in [13]. They are the oscillator wave function [17]

$$\begin{aligned}\phi_B^{(I)}(x) &= N\sqrt{x(1-x)} \exp \left[-\frac{m_B^2}{2\omega^2} \left(\frac{1}{2} - x - \frac{m_b^2}{2m_B^2} \right)^2 \right] \\ &\approx N\sqrt{x(1-x)} \exp \left[-\frac{m^2}{2\omega^2} x^2 \right],\end{aligned}\quad (34)$$

in our approximation $m_b \approx m_B$, and [18]

$$\Phi_B^{(II)}(x, \mathbf{k}_T) = N' \left[C + \frac{m_b^2}{1-x} + \frac{\mathbf{k}_T^2}{x(1-x)} \right]^{-2}, \quad (35)$$

where x is the momentum fraction of the light quark in the B meson. The parameters in $\phi_B^{(I)}$ are $\omega = 0.4$ GeV and $m = 5.28$ GeV. The constant N is determined by the normalization of the wave function:

$$\int_0^1 dx \phi_B^{(I)}(x) = \frac{f_B}{2\sqrt{3}}, \quad (36)$$

$f_B = 160$ MeV being the B meson decay constant [19], which leads to $N = 2.24$ GeV.

Similarly, the constants N' and C in $\Phi_B^{(II)}$ are determined by the following normalizations [18]:

$$\begin{aligned}\int_0^1 dx \int d^2\mathbf{k}_T \Phi_B^{(II)}(x, \mathbf{k}_T) &= \frac{f_B}{2\sqrt{3}}, \\ \int_0^1 dx \int d^2\mathbf{k}_T [\Phi_B^{(II)}(x, \mathbf{k}_T)]^2 &= \frac{1}{2},\end{aligned}\quad (37)$$

from which $N' = 1.232$ GeV³ and $C = -0.99993m^2$ are obtained. The Fourier transform of $\Phi_B^{(II)}$ gives

$$\begin{aligned}\phi_B^{(II)}(x, b) &= \int d^2\mathbf{k}_T \Phi_B^{(II)}(x, \mathbf{k}_T) e^{i\mathbf{k}_T \cdot \mathbf{b}} \\ &= \frac{\pi N' b x^2 (1-x)^2}{\sqrt{m^2 x + C x(1-x)}} K_1 \left(\sqrt{m^2 x + C x(1-x)} b \right),\end{aligned}\quad (38)$$

with K_1 the modified Bessel function of order one. As stated before, we neglect the intrinsic b dependence of the wave function, and define

$$\phi_B^{(II)}(x) = \lim_{b \rightarrow 0} \phi_B^{(II)}(x, b) = \frac{\pi N' x(1-x)^2}{m^2 + C(1-x)}. \quad (39)$$

Obviously, both of the models peak at small x , which signifies the soft dynamics in the B meson. However, the probability at intermediate x is indeed comparable at least in model II, and the resummation of double logarithms performed in section 3 is essential. At last, we adopt the Chernyak-Zhitnitsky model [20] for the pion wave function:

$$\phi_\pi(x) = 5\sqrt{3}f_\pi x(1-x)(1-2x)^2 \quad (40)$$

with $f_\pi = 93$ MeV the pion decay constant.

We are now ready to evaluate f_1 and f_2 numerically in eqs. (30) and (31). Results of f_i for the two models of ϕ_B , with b_1 and b_2 integrated up to the same cutoff $b_{1c} = b_{2c} = b_c$, are shown in fig. 7. It is found that at $\eta = 0.3$ approximately 50% of the contribution to f_i comes from the region where $\alpha_s(1/b_c) < 1$, or equivalently, $b_c < 0.5\Lambda$. At $\eta = 0.4$, 55% of the contribution is accumulated in this perturbative region. As $\eta = 1$, perturbative contribution has reached 70%. It implies that the PQCD analysis of the decay $B \rightarrow \pi l \nu$ in the range of $\eta > 0.3$ is relatively reliable according to the criteria given in [1, 4]. Therefore, we shall accept the modified PQCD predictions for $\eta \geq 0.3$, and use them to evaluate the differential decay rate. It is also observed that the expressions with $\phi_B^{(II)}$ employed are more perturbative, because the large k_1^- region is less suppressed by the wave function, and the double logarithms are stronger. The predictions from $\phi_B^{(I)}$ are larger, because $\phi_B^{(I)}$ enhances the contribution from the end-point region of x_1 .

In the approach of [13] where the transverse momentum dependence was not included, instead, energies of the virtual quark and gluon involved in the hard scattering were taken as the ultraviolet and infrared cutoffs of radiative corrections, respectively. The resulting Sudakov logarithm proportional

to $\ln m/k_1^-$, however, gives weaker suppression. From the steepest descent approximation of their Sudakov factor, the saddle point at k_1^- around 1.4Λ for $\eta = 1$ has been found. Using the same method, we determine the saddle point of our Sudakov factor in eq. (25) by

$$\frac{\partial S}{\partial b_1} = \frac{\partial S}{\partial b_2} = 0 , \quad (41)$$

from which a larger scale $1/b_1 = 10\Lambda$ for $\Lambda = 0.1$ GeV, or $1/b_1 = 6\Lambda$ for $\Lambda = 0.2$ GeV is obtained. It is then obvious that the perturbative expansion in our approach is more reliable. At this large scale the double logarithms contained in the $O(\alpha_s)$ radiative corrections to the hard scattering with a triple gluon vertex, which have been considered in [13], are in fact not important.

We observe that the magnitude of f_2 is much larger than that of f_1 , especially in the small η region. This fact is consistent with their behaviors in the soft pion limit as obtained in the framework of HQET [12], in which f_2 is found to have a pole at $\eta \rightarrow 0$:

$$\lim_{\eta \rightarrow 0} f_2 \approx \frac{2f_{B^*}}{\eta f_\pi} g_{BB^*\pi} , \quad (42)$$

for $m_{B^*} \approx m_B$, m_{B^*} being the B^* meson mass. In the above expression $g_{BB^*\pi}$ is the $BB^*\pi$ coupling constant, and f_{B^*} the decay constant of the B^* meson. Assuming $\eta = 0.3$, $f_{B^*} \approx 1.1f_B$ [21] and $g_{BB^*\pi} \approx 0.75$ [22] in eq. (42), we obtain $f_2 = 9.5$ which is close to our predictions at $\eta = 0.3$. Certainly, this extrapolation of the soft pion theorems may not be reliable, but it is interesting to observe that it is consistent with the PQCD results at the middle value of η . f_1 in the soft pion limit vanishes like $1 - \sqrt{m_{B^*}/m_B}$ [12].

With results of f_1 and f_2 , we can compute the differential decay rate of $B^0 \rightarrow \pi^- l^+ \nu$ with vanishing lepton masses [13]:

$$\begin{aligned} \frac{d\Gamma}{d\eta} &= |V_{ub}|^2 \frac{G_F^2 m^5 \eta^3}{768\pi^3} |f_1 + f_2|^2 \\ &\equiv |V_{ub}|^2 R(\eta) , \end{aligned} \quad (43)$$

for $\eta > 0.3$, where the second formula defines the function $R(\eta)$. Results of $R(\eta)$ are shown in fig. 8, from which the decrease of $d\Gamma/d\eta$ with η is observed. The behavior predicted here is opposite to that given in [13], which shows an increase in η starting with zero at $\eta = 0.5$. Such a dip at the middle value of η is attributed to the subtraction of an on-shell fermion propagator from the hard scattering [13], which is, however, not necessary in our treatment. Furthermore, the predictions in [13] for the differential decay rate are almost 10^3 times smaller than ours. The reason is again traced back to the subtraction of the fermion propagator. Therefore, the transverse momentum dependence plays an essential role in the analysis of B meson decays.

The differential decay rate in the soft pion limit can be obtained using eq. (42):

$$\lim_{\eta \rightarrow 0} R(\eta) = \frac{G_F^2 m^5 \eta f_{B^*}^2}{192 \pi^3 f_\pi^2} g_{BB^*\pi}^2, \quad (44)$$

which shows a linear relation with η . Extrapolating eq. (44) to intermediate η as shown in fig. 8, we observe a fair match between the soft pion and PQCD predictions around $\eta = 0.3$. It implies that the modified PQCD formalism is successful at large η , but becomes worse quickly in the soft pion limit. Similarly, the soft pion technique is appropriate at small η , but gives an overestimation to the considered process in the perturbative region. The overlap of these two approaches indicates the transition of the B meson decays to PQCD at middle values of η , and the complementarity between the soft pion theorems and modified perturbative formalism [7, 23].

We then estimate the total decay rate Γ by integrating $d\Gamma/d\eta$ using eq. (44) for $\eta < 0.3$ and using our PQCD predictions for $\eta > 0.3$. We obtain $0.6 \times 10^{-11} |V_{ub}|^2$ GeV from the soft pion theorems, and $1.9 \times 10^{-11} |V_{ub}|^2$ and $0.7 \times 10^{-11} |V_{ub}|^2$ GeV for the use of $\phi_B^{(I)}$ and $\phi_B^{(II)}$, respectively, from the modified PQCD formalism. Their sum gives $\Gamma \approx 2.5 \times 10^{-11} |V_{ub}|^2$ GeV for model I, and $1.3 \times 10^{-11} |V_{ub}|^2$ for model II. They correspond to branching

ratios $0.5 \times 10^2 |V_{ub}|^2$ and $0.26 \times 10^2 |V_{ub}|^2$, respectively, for the total width of the B^0 meson is $(0.51 \pm 0.02) \times 10^{-9}$ MeV [24]. Current experimental limit on the branching ratio of $B^0 \rightarrow \pi^- l^+ \nu$ is 3.3×10^{-4} [25]. We then extract the the matrix element $|V_{ub}| < 2.6 \times 10^{-3}$ from model I and $|V_{ub}| < 3.6 \times 10^{-3}$ from model II. These upper limits are close to the value 0.003 given in the literature [24].

5. Conclusion

In this paper we have applied the resummation technique to the semi-leptonic decay $B \rightarrow \pi l \nu$, and derived the Sudakov factor up to next-to-leading logarithms in this heavy-to-light process. The idea is to apply the eikonal approximation to the heavy b quark line such that its nonleading self-energy diagram is excluded, and the number of large scales involved in the B wave function is reduced. The resummation of double logarithms in the heavy meson is then simplified to the one in analogy with the light meson. The modified PQCD calculation of the differential decay rate including Sudakov effects has been examined and found to be reliable for η above 0.3. By combining our predictions with the soft pion results and comparing them with experimental data, we have estimated the total decay rate, from which the upper limit 0.003 for the KM matrix element $|V_{ub}|$ is obtained.

We do not observe the dip at $\eta = 0.5$ for the differential decay rate as predicted in [13], which arises from the subtraction of an on-shell fermion propagator from the hard scattering. This subtraction is not necessary in our analysis because of the inclusion the transverse momentum dependence. The behavior in η is similar to that of the pion form factor in Q^2 , which is opposite to the results of [13]. We do not consider the evolution in b and the intrinsic transverse momentum dependence of wave functions in this

work. However, we estimate that these two modifications cancel at least partially, since the former gives an enhancement [4] to the decay rate, but the latter leads to a suppression [15]. Certainly, this subject needs more detailed investigation. Our formalism can be easily applied to a similar semi-leptonic decay $B \rightarrow \rho l \nu$ [13] and other non-leptonic B meson decays, which will be published elsewhere.

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Figure Captions

Fig. 1. $O(\alpha_s)$ radiative corrections to the basic factorization of the pion form factor.

Fig. 2. Graphic representation of eq. (5).

Fig. 3. Lowest-order factorization for the decay $B \rightarrow \pi l \nu$.

Fig. 4. $O(\alpha_s)$ radiative corrections to the B meson wave function.

Fig. 5. Eikonal approximation for the b quark line.

Fig. 6. Lowest-order diagrams for (a) the function \mathcal{K} and (b) the function \mathcal{G} associated with the B meson.

Fig. 7. Dependence of (a) f_1 and (b) f_2 on the cutoff b_c derived from $\phi_B^{(I)}$ (solid lines) and from $\phi_B^{(II)}$ (dashed lines) for (1) $\eta = 0.3$, (2) $\eta = 0.4$, and (3) $\eta = 1.0$.

Fig. 8. Dependence of $R(\eta)$ on η derived from (1) $\phi_B^{(I)}$ and from (2) $\phi_B^{(II)}$ (solid lines). Results from the soft pion theorems are also shown (dashed line).

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